

A REVIEW OF WAVE MAKERS FOR 3D NUMERICAL SIMULATIONS

Pál Schmitt *, Björn Elsaesser*

*Marine Research Group,
Queen's University Belfast,
BT9 5AG Belfast,
Northern Ireland
p.schmitt@qub.ac.uk

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Abstract. Numerical methods have enabled the simulation of complex problems in off-shore and marine engineering. A significant challenge in these simulations is the creation of a realistic wave field. A good numerical tank requires wave creation and absorption of waves at various locations. Several numerical wavemakers with these capabilities have been presented in the past. This paper reviews four different wave-maker methods and discusses limitations, computational efficiency, requirements on the mesh and preprocessing and complexity of implementation.

1 INTRODUCTION

While numerical methods enable the simulation of complex flow phenomena, like two phase flows and gravity waves, to an increasing extend, when setting up a simulation of coastal or marine engineering processes a recurring issue is the creation and absorption of waves.

A wide range of options are now available in the literature but broadly they can be divided into two main categories. Wave makers are either simulated through the implementation of a physically similar wave generator or through numerical/mathematical implementation either as source terms or similar. One such method relating to the first category is the direct simulation of the wave-maker by a moving mesh. This is very similar to what happens in a laboratory tank but is computationally expensive and also comes with all the issues of physical wave makers (i.e. absorption, evanescent wave, piston vs hinge motion). The mesh motion can be a complex task and at least a basic wave maker control theory must be implemented, just as it would be in an experimental facility.

More efficient than mimicking the physical equivalent directly, is the use of numerical algorithms to create the wanted flow conditions. Numerical methods (second category)

offer the possibility to manipulate field variables. For the purpose of this paper the focus is on simulations using the Volume of Fluid (VoF) method, although most methods could well be applied to other numerical methods. It is assumed that the species α (air or water), velocity U and pressure p exist as variables in these simulation and the flow is discretised in full three dimensions, as opposed to shallow water/Boussinesq equations, for example. Furthermore, it is assumed that the simulations are set up correctly to model the behaviour of the fluid, such that any disturbance will result in real gravity waves. The difficulty is then, to create waves of a specified height and period, while at the same time absorbing the waves reflected inside the tank.

In most cases the generation of the wave is not of primary interest in the numerical model, it simply is a means of simulating the effect of waves away from the wave-maker, where the main computational effort should be concentrated. Thus a solution that is as little computationally demanding as possible is desired.

In principle all methods in the numerical wave generation category can be grouped into four types, these will be discussed in the following sections.

- *relaxation method* - this type relaxes the results of the simulation inside the domain with the results given by wave theory (linear or higher order)
- *mass source function* - the wave is created by adding a source term to the continuity equation
- *impulse source function* - the wave is created by adding a source term to the impulse equation
- *boundary method* - while the above three methods operate inside the computational domain, a fourth method is discussed that creates and absorbs waves on boundary patches.

This paper will discuss the advantages and disadvantages without detailing the actual implementation of any code, while focussing on the mathematical or computational model. The choice of a suitable wave-maker might depend on

- speed of simulation/computational burden
- availability/complexity of implementation
- requirements on preprocessing
- accuracy/ease of calibration
- range of wave parameters

Some features, like the ease of calibration, might be of most importance for example when trying to recreate experimental tank data. Calibrating the waves may take several simulations, hence a more accurate but slightly slower wave-maker might be advantageous.

In order to deal with waves reflected back towards the wave-maker, the first three types above rely on independent means of absorption of reflected waves. Thus numerical beaches are discussed in the following section, followed by descriptions and discussion of the wave-makers.

2 NUMERICAL BEACH

Physical testing facilities often use sloped bottom and energy absorbing materials like gravel or wire meshes to absorb waves at the end of the tank. In analogy to that, a wave absorbing method can be implemented in the numerical wave tank and is called a numerical beach in the remainder of this document.

The beach is often modelled by implementing a dissipative source term in the impulse equation. For example [2] used an implementation in which the dissipation acted only on the velocity component in gravity direction. In this paper a term acting equally in all directions is used. The dissipation parameter \mathbf{s} can be set as a field variable to model the beach and has no effect where set to zero.

According to [3] the beach is most effective if the variable \mathbf{s} is set as a cubic function of the spatial variable x and has soft entries. The following function is used here to set the variable \mathbf{s} :

$$\mathbf{s}(\mathbf{x}) = -\frac{2 \cdot \mathbf{s}_{\max}}{(x_{\text{end}} - x_{\text{start}})^3}(x - x_{\text{start}})^3 + \frac{3 \cdot \mathbf{s}_{\max}}{(x_{\text{end}} - x_{\text{start}})^2}(x - x_{\text{start}})^2 \quad (1)$$

where \mathbf{s}_{\max} is the maximum value of the dissipation parameter, x_{start} and x_{end} are the starting and end points of the spatial extension of the beach.

The value of \mathbf{s} has no physical meaning; to find optimum settings numerical studies were performed to analyse the effect of different values for the beach length L_{beach} and dissipation parameter. Results are presented in Fig. 1. The length of the beach is varied between half and three times the wave length λ . The variable \mathbf{s} is tested for maximum values between one and ten, the distribution used is described by equation 1. Figure 1 shows the ratio of reflected to incoming wave amplitude A_{ref}/A , evaluated according to the method presented by [8], for different beach to wave length λ ratios and \mathbf{s}_{\max} values. It can be seen that almost full reflection occurs for a beach of half the wave length and an \mathbf{s}_{\max} value of 1. Reflection coefficients decrease sharply for the same beach length for increasing values of \mathbf{s}_{\max} , to a minimum of 0.1 for a \mathbf{s}_{\max} value of 5. For larger values the reflection increases again. A longer beach always decreases reflection, for a length of approximately three wave lengths reflection coefficients are less than 0.1 over almost the whole range of tested \mathbf{s}_{\max} values. A beach lengths of more than half a wavelength is needed to achieve reflection coefficients of less than 0.1, with one wave length the reflection coefficient becomes almost zero. Values for \mathbf{s} should be above 2.5, with an optimum absorption for values around 5.

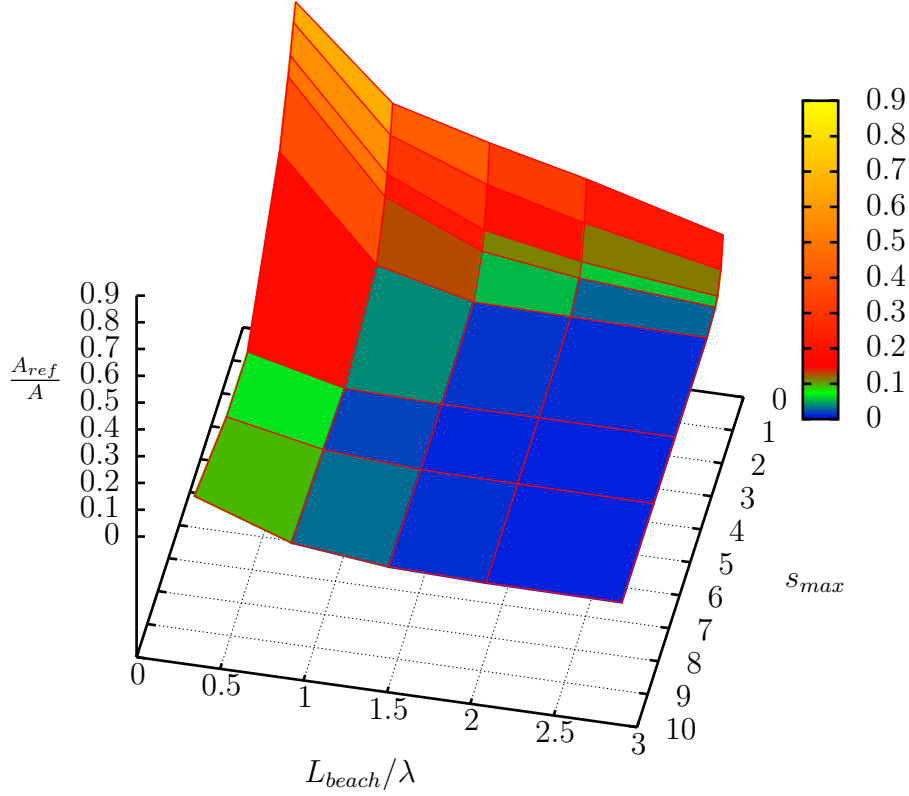


Figure 1: Amplitude reflection coefficients x_{ref} for different beach configurations.

It can thus be safely assumed that a beach with a length of approximately one wave length and a value of 5 for the s_{max} provides very good absorption ($A_{ref}/A < 0.1$). Figure 2 gives an overview of the numerical tanks employing different wave makers. The numerical beach described here is used in the first three set-ups and shown in yellow.

3 RELAXATION METHOD

Similar to the under-relaxation method commonly used in solving systems of equations [4], waves can be generated in a domain by blending or *relaxing* the values in the domain with a given value at every time-step. In the method presented by [1] values for velocity,

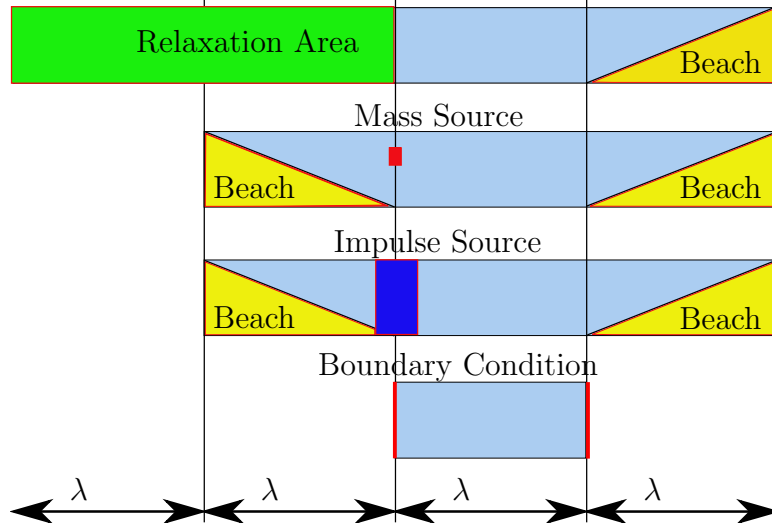


Figure 2: Sketch with set up of numerical tanks for different wave-makers.

species and pressure are updated every time-step according to following formula:

$$U = c_{relaxation}U_{ana} - (1 - c_{relaxation})U \quad (2)$$

$$\alpha = c_{relaxation}\alpha_{ana} - (1 - c_{relaxation})\alpha \quad (3)$$

$$p = c_{relaxation}p_{ana} - (1 - c_{relaxation})p \quad (4)$$

where the index *ana* stands for the fields containing the analytical solution and $c_{relaxation}$ is the relaxation coefficient, taking values between 0 and 1.

Depending on the value of $c_{relaxation}$, the result of the simulation is completely overwritten by the analytical solution, blended or left unchanged. It is thus possible to impose any wave at any position but let the simulation obtain results around an object of interest. The blending allows for reflected waves to be gradually smoothed out.

This simple method eliminates all problems associated with absorption of incoming waves in the wave-maker and disturbance of incoming waves. If the wave is imposed all around the object an open sea conditions can easily be simulated in relatively small domains, provided the bottom is not sloped or the imposed wave takes into account the changing depth. [1] used one wavelength to ramp the wave up and a minimum of one additional wavelength to blend the wave into the simulation domain. Tests by the author showed that in many cases good results are achieved even if the blending length is less than one wavelength. Setting the field values and blending requires iterating over all of the cells in the wavemaker. The computational cost is thus rather high. More importantly, it might often be unnecessary to impose an analytical solution on the pressure, since the pressure is typically obtained from the velocity field in the pressure correction loop in many solvers. Not imposing the pressure saves approximately 30% of execution time, depending on wave theory and formulation.

This type of wave-maker was used successfully for the simulation of fixed flap structures as presented in [10]. Figure 2 shows a sketch of a numerical wave tank employing the relaxation method. The relaxation zone stretches over two wavelengths λ . A numerical beach is added at the end of the tank. With a testing area of one wavelength the numerical wave tank extends over four wavelengths.

Imposing the velocity leads to a locally confined divergence of the pressure at the center of the wave maker region, which does not affect the results in the domain as long as the local Courant number is relatively large. For simulations of moving bodies, and resulting small time steps, the author found the method to diverge.

3.1 MASS SOURCE FUNCTIONS

This method is based on the idea that displacement off the surface can be induced by the inflow and outflow of water into the domain. [7] implemented a source function by modifying the continuity equation, the extension of [5] uses a vertical inlet velocity on one face of a cell prescribing the source volume. All source type methods are transparent, meaning a wave will pass the wavemaker without interaction. They can thus be used in front of a numerical beach to create absorbing wave-makers as shown in Figure 2.

The basic formulation is the relation between surface elevation $\eta(t)$, wave celerity c , source area A and source function $s(t)$. The number 2 stems from the fact that *two* waves propagate from the source region, one on each side in a two dimensional case.

$$s(t) = \frac{2c\eta(t)}{A} \quad (5)$$

Transforming the source function into a surface flux $q(t)$ results in:

$$q(t) = \int_V s(t)dV = s(t)V = \frac{2c\eta(t)V}{A} = 2c\eta(t)dy \quad (6)$$

The flux can be translated into a velocity U_n normal to the patch surface A_s to be used as a boundary condition for the velocity:

$$U_n = \frac{q(t)}{A_s} \quad (7)$$

Alternatively the continuity equation in the pressure correction procedure can be adapted with a volume source term:

$$\nabla \mathbf{U} = S_m \quad (8)$$

This implementation only works correctly if the wave maker is defined over the whole width of the tank and has a constant cross section area A_s .

This method is well suited for the implementation in codes for a number of reasons.

1. Any surface deformation or wave type $\eta(t)$ can simply be implemented in a single framework.

2. No evaluation of the particle velocities needs to be performed, reducing computational burden.
3. No iteration over the faces in the boundary patch is necessary. All faces are multiplied with the same velocity value.

The wave maker itself emits only waves, reflected waves cross the wave-maker region without interaction. To absorb reflected waves a beach must be located before the wave-maker, as shown in Figure 2.

For limited wave heights this type of wave maker yields excellent results. If the recommendations of [7] for the size and position of the wave-maker region are observed the target wave is created with high accuracy, thus no calibration is required. It should also be noted that the size of the wave-maker region is very small. Imposing the velocities only requires iterations over few cells or faces.

A major drawback for the application to large waves in shallow water are the requirements on placement and size of the wave maker region. If the water surface reaches the wave maker region the simulation fails. It should be noted that the surface elevation at the wave maker position varies about twice as much as the wave height of the target wave. Creating a wave with a height of more than about 10% the water depth seems impossible, since the wave maker region must be placed above half the water-depth and extend over at least 1/5 the water-depth [7].

[9] also describes the formation of vortices below the mass source region in deep water. However, this does not seem to have had an effect on the simulation of a nearby floating body.

3.2 IMPULSE SOURCE FUNCTIONS

As an extension, or logical further development, of the mass source term method it is also possible to define an impulse source term [2]. Adding a source term to the momentum equation does not seem to limit the wave height in any way, otherwise the application is very similar to the *mass source* method. The author's implementation uses a source term defined as the product of density ρ , the scalar field defining the wave-maker region \mathbf{r} and the analytical solution of the wave velocity \mathbf{U}_{ana} at each cell centre yielding the adapted impulse equation:

$$\frac{\partial(\rho\mathbf{U})}{\partial t} + \nabla(\rho\mathbf{U}\mathbf{U}) = -\nabla p + \nabla\mathbf{T} + \rho\mathbf{f}_b + \mathbf{r}\rho\mathbf{U}_{ana} \quad (9)$$

This implementation allows the generation of waves up to the limit of breaking and also the specification of a propagation direction. Depending on the implementation, the analytical solution for the local velocity in each cell or at least the depth averaged velocity must be obtained and applied to all cells over the height of the water column. The computational burden is likely to be higher than for the mass source method, since values need to be set over a considerable number of cells, as shown in Figure 2.

In the original work by [2] the source terms are derived from the linear Boussinesq equations and the same velocity value is applied over the entire height. Applying a single velocity value over the height of the water column will decrease the computational burden, compared to applying spatially varying velocity values according to a wave theory of choice. It was initially believed to be more accurate to use the exact analytical solution but in cases where extreme waves are created the resulting wave height was up to 20% lower than expected. Considering that the analytically prescribed velocities reach their maximum at the surface, these highest velocities are not applied to the water and the missing impulse will inhibit the wave building up to the specified height. It might thus be that using one average value over the wave height leads to more accurate waves.

4 BOUNDARY CONDITIONS

A fairly simple approach is to prescribe the wave velocity and water height as an inflow boundary condition, as shown in Figure 2. This method was tested by the author but was found to be very sensitive to the wave theory applied. Stability problems occur when the water-level and the velocity are prescribed. Little improvement in stability can be achieved by only prescribing the velocity field. The free surface height is moved by the velocity field and thus waves can be created.

Absorption of reflected waves is still a challenge and only recently [6] seem to have overcome the problems and successfully implemented an absorbing wave-maker that works directly on a boundary patch. Waves are created by prescribing the velocity and surface elevation on the patch according to a theoretical wave theory of choice or assumed piston wave-maker motion simulation. The main challenge of absorbing waves is solved by applying a correction velocity U_{corr} . U_{corr} is derived from the difference between the target surface elevation and the actual elevation η_r . If U is the horizontal vertically integrated (uniform) velocity and c is the wave celerity, the water-depth h and deviation from that mean surface η are related by the following equation:

$$Uh = c\eta \quad (10)$$

Assuming linear shallow water waves, the celerity can be approximated as

$$c = \sqrt{gh}. \quad (11)$$

Substituting c in Equation 10 yields a simple formula for the velocity correction on the patch to remove a reflected wave of height η_r :

$$U_{corr} = \sqrt{\frac{g}{h}}\eta_r \quad (12)$$

It should be noted that this correction is constant over the height of the patch. The method also requires measuring the surface elevation accurately, increasing the complexity of the implementation. [6] also presented advanced absorption methods for three dimensions and showed in test cases that amplitude reflection coefficients of below 10% are

readily achieved. As shown in the overview in Figure 2, the boundary conditions can be used directly as a boundary to the test section, reducing the size of the numerical wave tank to the bare minimum.

In a three dimensional test case, including a strongly reflecting moving body, the author found this boundary method to be roughly 10% faster compared to the momentum source method discussed above. In an empty tank, without local mesh refinement around a body for example, the runtime is expected to scale with the mesh size, so the gains in computational efficiency of this wave maker would be expected to be approximately 50 – 60%. No problems with reflected waves were encountered in this case, but more data are needed for a representative comparison. It is also not clear whether the linearisation employed in Equation 11 causes issues in deep water conditions or how the method could best be adapted for such cases.

5 CONCLUSIONS

This paper presented four numerical wave makers. All methods discussed here are able to absorb reflected waves, either by active wave absorption or by coupling with a numerical beach.

The *relaxation* method was found to be numerically unstable due to the imposed mismatch of pressure and velocity and have a fairly high computational burden.

Mass source methods are easy to implement, efficient and produce waves accurately without calibration. However they can fail in extreme waves, especially in shallow water.

Impulse source functions are somewhat more computationally expensive and complex to implement, but not limited in wave height.

The *boundary method* allows for the smallest computational domain of all methods, since no numerical beach is required. Absorption and wave generation is complex to implement, but in principle should reduce the runtime compared to the impulse source function. This function also only works in shallow water at present.

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